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MAGNETIC EFFECT ON CASSON'S FLUID FLOW IN SLIP REGIME

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ABSTRACT
A flow of incompressible fluid through a tube under the influence of magnetic field has been considered to find shear stress using an iterative method. Due to independent nature of shear stress on fluid, this shear stress has been used to analyze the flow of Casson's fluid in slip regime. The velocity field and volume flow rate have been computed through trapezoidal rule. The influence of magnetic field on shear stress has been shown graphically. The variation of velocity has also been seen by graphs for different values of yield stress and different values of magnetic field parameter. The nature of volume flow rate has also discussed with the help of graphs.

Keywords: Magnetic Effect**,** Casson's Fluid, Slip Regime, Blood Flow, Artery.

1. INTRODUCTION

The flow characteristic of Casson fluid in tubes was first investigated by Oka. He considered a generalized model for flow of non-Newtonian fluid in a tube from which the Casson fluid model was derived as a special case. Oka[12] studied blood flow in capillaries with permeable wall using the Casson model. The shear stress and shear rate relation given by Casson satisfactory describes the properties of many polymers over a wide range of shear rates. Casson fluid is found to be applicable for developing a model for blood and haemodilysers [11]. In 1958, Casson basing his calculations on certain assumptions concerning the chain like floccules presents in varnishes and printing inks derived this semi-empirical equation to describe their flow behavior.

A year later, Scott Blair in 1959 found that this description was also suitable for the flow of blood. Since then, there have been various studies made supporting the use of Casson equation to blood flow. The details of the study for fully developed and laminar flow of Casson fluid have been described by Fung [6], later Chaturani and Ranganathan [2] extended Oka's work considering axial variation of viscosity. Consideration of entrance region of blood as a Casson fluid was attempte numerically by Shah and Soto [16]. Among the existing models to describe the constitutive behavior of blood, two equations are most popular, one is power law and other is Casson equation. Each model has its own merits, Merill et. al in 1963 found that the measured data were consistent with Casson equation at shear rates 0.1 to 1 $\sec^{-1}$, but deviated to some extent in the range of 1 to 40 $\sec^{-1}$ (Milnor, 1982). Bate(1977), believed that the blood flow through tubes on best described by casson equation in the shear range of 15 to 6400 $\sec^{-1}$ [7].

Blood, a class of fluid, has its components as plasma, platelets, erythocytes and leukocytes. But most of its rheological properties depends on RBC which is 40-50% of the total blood volume [15] under normal conditions. It is generally assumed that the blood plasma does not obey the slip condition the presence of RBC moving close to the wall may be through to be equivalent to a continuum which is sliping at the wall. At the boundary, a slip in axial velocity and a particle rotating depending upon wall effect parameter. Authors [1, 3, 5, 9, 10, 15] have used slip at the wall as a mechanism in explaining the blood flow.

In the present paper, we have to study the effect of magnetic parameter on blood flow represented by Casson model considering slip condition at the wall. We consider the flow of incompressible fluid through a tube under the influence of magnetic field to find shear stress using iterative method. Due to independent nature of shear stress on fluid, we use this shear stress to analyze the flow of Casson's fluid. Using trapezoidal rule, we compute velocity field and volume flow rate. The influence of magnetic field on shear stress has been shown graphically. The variation of velocity has also been seen by graphs for different values of yield stress, magnetic parameter and slip parameter. The similar study has also been carried forward for volume flow rate through graphs.

2. FORMULATION OF THE PROBLEM

Consider steady, laminar and axial symmetric flow of blood through an artery under the influence of a externally applied homogeneous magnetic field. The blood flow in the tube is assumed to be a suspension of red cells in plasma. It is also assumed that the density and viscosity are constant and the electromagnetic force produced is very small. Under the assumption of the small electrical conductivity, the one dimensional, simplified momentum equation for the flow due to Haldar [8] is

$$-\frac{dp}{dz}+\frac{1}{r}\frac{d}{dr}\left(\mu r\frac{du}{dr}\right)+B_0^2\,\sigma_e u=0,\ 0\le r\le a \tag{1}$$

where μ is the viscosity, u is the axial velocity, p is the pressure, $\beta_0=\mu_e H_0$ is the electromagnetic induction, μ_e is the magnetic permeability, H_0 is the intensity of magnetic field , σ_e is the conductivity of fluid and a is radius of the tube like an artery.

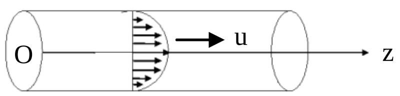

Fig 1. Velocity profile for Hagen- Poiseuille flow

We know that by Newton's law of viscosity [8]

$$\tau=-\mu\frac{du}{dr} \tag{2}$$

where τ and μ are the shear stress and coefficient of blood viscosity respectively.

The relevant boundary conditions are

$$u=-L\frac{du}{dr}\quad at\quad u=a \tag{3}$$

$$\frac{du}{dr}=0\quad at\qquad r=0 \tag{4}$$

where L is the slip coefficient.

In terms of shear stress τ , the equation (1) can be written as

$$\frac{d}{dr}(\tau r)-\frac{\mu_e^2 H_0^2\sigma_e\, r}{\mu}\left\{\int_r^a \tau dr+L\tau\right\}=-r\frac{dp}{dz},\ 0\le r\le a \tag{5}$$

Since the shear strain distribution in pipe or tube is independent of the type of the fluid so the equation (5) is valid for any kind of fluid.

Introducing the following dimensionless parameters

$$u^*=\frac{u}{u_c}\ ,\ r^*=\frac{r}{a}\ ,\ \tau^*=\frac{\tau}{\tau_c} \tag{6}$$

$$where\ \ u_c=\frac{-a^2}{2\mu}\left(\frac{dp}{dz}\right)\ and\ \ \tau_c=\frac{\mu}{a}u_c$$

Incorporating all these dimensionless variables in the equation (5), we have

$$\frac{d}{dr^*}\left(\tau^* r^*\right)-M^2 r^*\int_{r^*a}^{a}\tau^* dr^*-\frac{M^2 L}{a}\tau^*=2r^* \tag{7}$$

where $M^2=\frac{\sigma_e H_0^2\mu_e^2 a^2}{\mu}$ is the Hartmann number.

The reduced boundary conditions and Newton's law of viscosity are

$$u^*=-L\frac{du^*}{dr^*}\quad at\quad r^*=1 \tag{8}$$

$$\frac{du^*}{dr^*}=0\quad at\qquad r^*=0 \tag{9}$$

$$and\quad \tau^*=-\frac{du^*}{dr^*} \tag{10}$$

Ignoring the star for our convenience, we have

$$\frac{d}{dr}(\tau r)=2r+\frac{M^2 L}{a}\tau+M^2 r\int_r^1 \tau dr \tag{11}$$

and the boundary conditions are

$$u=-\frac{L}{a}\frac{du}{dr}\quad at\quad r=1 \tag{12}$$

$$\frac{du}{dr}=0\quad at\qquad r=0 \tag{13}$$

$$\tau=-\frac{du}{dr} \tag{14}$$

Integrating equation (11), we get

$$\tau=r+\frac{M^2 L}{a}\frac{1}{r}\int_0^r \tau dr+\frac{M^2}{r}\int_0^r r\left(\int_r^1 \tau dr\right)dr,\ 0\le r\le 1 \tag{15}$$

Here to solve double integral in equation (15), we apply the following iterative scheme:

$$\tau_j^{(n+1)}=r_j+\frac{M^2 k}{r_j}\int_0^{r_j}\tau_j^n dr+\frac{M^2}{r_j}\int_0^{r_j} r_j\left(\int_{r_j}^1 \tau_j^n dr_j\right)dr_j \tag{16}$$

where $n=0,1,2,3,...............$

$r_j=jh,\quad j=0,1,2,3,......N$,

$N=1/h$, $k=L/a$ is a slip parameter

where h is the step length in the radial direction and N is the total number of the radial nodal points. The iteration is continued as long as satisfies the condition

$$Max\,\tau_j^{(n+1)} - \tau_j^{(n)} \le \varepsilon$$

where ε is a pre-assigned error of order 10^{-4} of tolerance. The initial guess $\tau_j^{(0)} = 0$ is taken at each $j = 0,1,2,3,.....N$. This iterative scheme is found to be convergent. To evaluate the flow field completely, τ must be supplemented by a constitutive equation relating shear stress and shear rate due to Das, Mehta and Jayaraman [4].

3. CASSON'S CONSTITUTIVE EQUATION

Casson's constitutive equation in time-independent shear flow is a non-linear relation between shear stress and shear rate of strain [4]. It is given by

$$\tau^{\frac{1}{2}} = \tau_y^{\frac{1}{2}} + \left[-\mu\frac{du}{dr}\right]^{\frac{1}{2}},\ if\ \tau \ge \tau_y\ and\ \frac{du}{dr} = 0,\ if\quad \tau \le \tau_y \qquad (17)$$

τ_y denotes the yield stress and μ denotes Casson's velocity (viscosity at high shear rate). Equation (17) is valid for any kind of fluid, hence for blood, the shear stress acting on any cylindrical surface ($r =$ constant) is proportional to r on the wall.

In non-dimensional form, the Casson's constitutive equation (17) becomes

$$\tau^{*\frac{1}{2}} = \left(\frac{\tau_y}{\tau_c}\right)^{\frac{1}{2}} + \left(-\frac{du^*}{dr^*}\right)^{\frac{1}{2}},\ if\ \tau^* \ge \theta$$

$$and\quad \frac{du^*}{dr^*} = 0,\ if\quad \tau^* \le \theta \qquad (18)$$

where $\theta = \tau_y / \tau_c$ is the dimensionless yield stress, ignoring the star, we get

$$\tau^{\frac{1}{2}} = \theta^{\frac{1}{2}} + \left(-\frac{du}{dr}\right)^{\frac{1}{2}},\ if\ \tau \ge \theta\ \ and\ \ \frac{du}{dr} = 0,\ \ if\quad \tau \le \theta \qquad (19)$$

4. SOLUTION FOR VELOCITY DISTRIBUTATION

Integrating equation (19) from r to 1 with respect to r, using the boundary condition u=0 at $r = 1$, we get

$$u = \int_r^1 \tau\, dr + \int_r^1 \theta\, dr - 2\sqrt{\theta}\int_r^1 \sqrt{\tau} dr + \frac{L}{a}\tau,\ r_p \le r \le 1$$

(20)

where μ is the constant blood viscosity and r_p is the plug flow radius.

Now the equation (20) may be written as in iterative form

$$u_j(M,k) = \int_{r_j}^1 \tau(M,k)dr + \theta(1-r_j) - 2\sqrt{\theta}\int_{r_j}^1 \sqrt{\tau(M,k)}dr + \tau_j(M,k) \qquad (21)$$

Since τ is known at radial points only, it is convenient to evaluate the integral equation (21) using the trapezoidal rule. The velocity in the plug flow region is given by

$$u_j = u_p \quad for \quad r_j = r_p,\quad 0 \le r_j \le r_p \qquad (22)$$

5. FLOW RATE

The non-dimensional flow rate can be computed as

$$Q = 8\int_0^1 ur\, dr \qquad (23)$$

Equation (23) can be written in iterative form as

$$Q_j(M,k) = 8\int_{r_j}^1 u(M,k) r\, dr \qquad (24)$$

and using trapezoidal rule, for j=0 and M=0, 2, we get the required results.

6. DISCUSSION AND RESULTS

In the course of analysis of the problem of this paper, we come across the following results:

(1) In figure 2 and 3, the effect of slip parameter k has been studied in the presence of magnetic field M. It is our pleasure to state that magnetic field and slip parameter both have the impact on shear stress. We summarize our observations as the following:

(a) Figure 2 shows that shear stress remains constant for all values of slip parameter k we mean that introduction of slip at the wall of artery in the absence of magnetic field does not do any work which we may be considered.

(b) In figure 3, we see the effect of slip parameter k. As slip at wall of artery increases in the presence of magnetic field, shear stress τ increases. As slip parameter is increasing shear stress is going to be strong and it may be beneficial to remove the deposition of any fat or salt on the wall of artery. Thus, the introduction of slip at the walls works only in the presence of magnetic field. Practically it should work in the absence of magnetic field but as per our model it does not.

(2) In figure 4 and 5, the effect of slip parameter on velocity field u has been shown. We have the following conclusions:

(a) In the absence of magnetic field $M = 0$, slip parameter accelerates the velocity field. Referring figure 4, we see that the velocity is constant and it has not been affected by slip parameter. The effect of slip parameter gradually accelerates the flow as we moves away from axis of artery towards the wall. Overall, slip parameter has uniform effect on velocity as its value increases.

(b) In figure 5, we are seeing the remarkable effect of both magnetic field M and slip parameter k on velocity field. Along the axis of artery u_0 is constant, so there is no effect of any one of M and k. Under the influence of slip parameter k, velocity increases with the increase of radial distance while M accelerates the velocity in a good amount. Thus, this study again supports that the introduction of slip at wall in the presence of magnetic field may remove the formation of stenosis. Seeing the figures 4 and 5 we come to the point of convergence of velocities. This point convergence shows the matching position of free velocity and velocity due to the introduction of slip at the wall.

(3) The effect of magnetic parameter M and of slip parameter k on the volume flow rate has been depicted in figure 6 and 7 we observe that

(a) Referring figure 6, we notice that there is no effect on flow rate of slip parameter k in plug flow region. This fact has been represented by Q_0 . Thus in the absence of magnetic field, slip parameter has no control on volume flow rate in plug flow region.

(b) In presence of magnetic field, there is effect of magnetic field as well as slip parameter. Magnetic field accelerates nicely the volume flow rate. Hence, adjusting magnetic parameter, one can regulate volume flow rate. Looking upon the figure 7, there is a nice impact of magnetic and slip parameter beyond $k=0.2$. Thus, introduction of slip parameter in the presence of strong magnetic field, a good amount of fluid (blood) may be supply to heart and it may be helpful in hypertension cases.

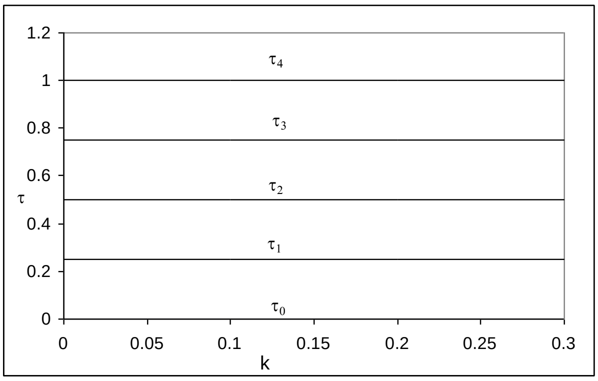

Fig 2. Effect of slip parameter k on shear stress τ for $M = 0$ (no magnetic field)

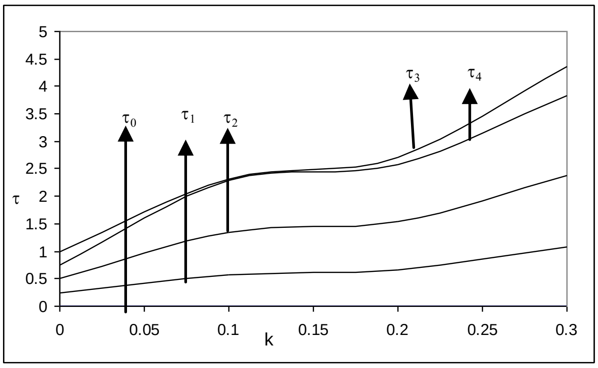

Fig 3. Effect of slip parameter k on shear stress τ for $M = 2$

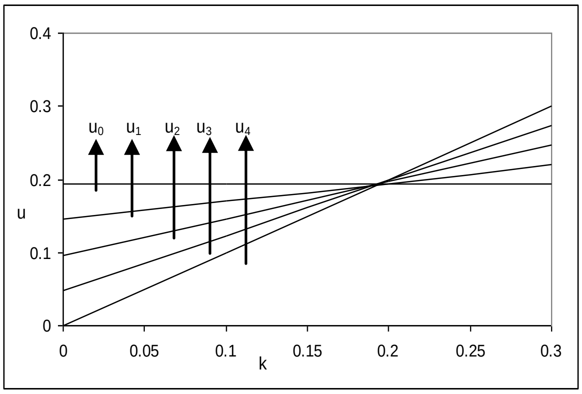

Fig 4. Effect of slip parameter k on velocity u for $M = 0$ (no magnetic field)

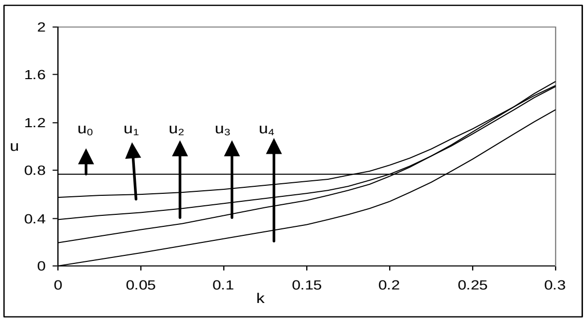

Fig 5. Effect of slip parameter k on velocity u for $M = 2$

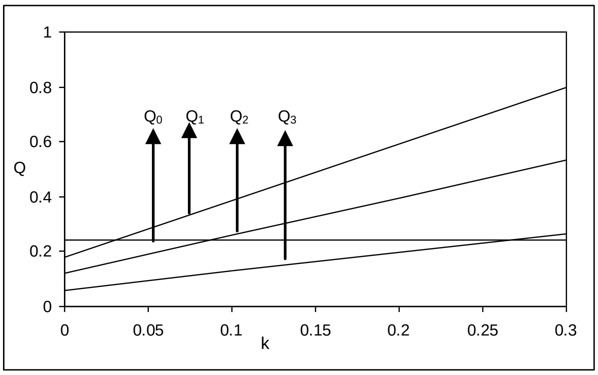

Fig 6. Effect of slip parameter k on volume flow rate Q for $M = 0$ (no magnetic field)

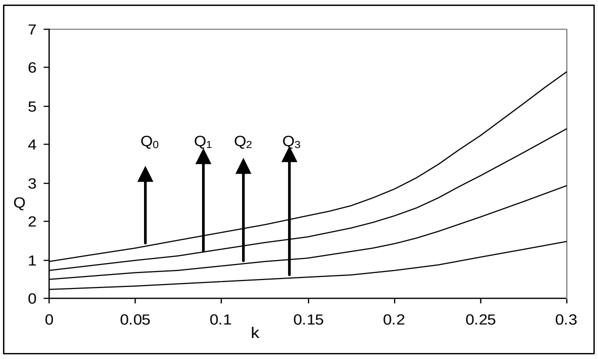

Fig 7. Effect of slip parameter k on volume flow rate Q for $M = 2$

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